Bonus-Malus pricing with dependent claim frequency and severity

Weihong Ni [1] & Eric C.K. Cheung [2] & Jae-Kyung Woo [3]

[1] *Department of Statistics and Actuarial Science, The University of Hong Kong, wni.grace@gmail.com*
[2] *Department of Statistics and Actuarial Science, The University of Hong Kong, eckc@hku.hk*
[3] *Department of Statistics and Actuarial Science, The University of Hong Kong, jkwoo@hku.hk*

Résumé. Dans le cadre Bonus-Malus, les prix reposent souvent sur le réajustement de la prime a posteriori, dans le cas où les primes individuelles pourraient être ajustées en fonction des antécédents personnels. L'estimation bayésienne joue normalement un rôle clé lors de la mise en œuvre d'une telle procédure. Dans la littérature, une hypothèse importante est l'indépendance entre la fréquence des sinistres et le montant des sinistres (ou sévérité). Ces deux variables sont souvent considérées séparment lors de la réalisation de l'ajustement du modèle et des estimations des paramètres. Dans la suite des résultats obtenus dans Ni et al. (2014), nous introduisons dans ce travail, la fréquence et la sévérité sont liées par un prior conjugué bivarié de la famille exponentielle linéaire. Nous en déduisons une répartition a posteriori bivariée pour les paramètresrelatifs au numéro et la taille de la demande, ce qui nous permet en plus de calculer une prime nette estimée. Un exemple concret sera également présenté avec le prior donné par une distribution mélange d'Erlang. Nous utilisons également un ensemble de données d'assurance automobile pour vérifer d'abord la dépendance entre la fréquence et la sévérité, puis pour ajuster le modèle et calculer le montant de la prime nette numériquement. En conclusion, une comparaison avec le cas indépendant sera illustrée théoriquement et numériquement.

Mots-clés. Modles pour lassurance et la finance, Mthodes baysiennes, Bonus-Malus, Conjugu bivari antrieur, Bivari mixte Erlang

Abstract. Under a Bonus-Malus framework, pricing often relies on the a posteriori premium adjustment where individual premiums could be adjusted according to personal claim histories. Bayesian estimation normally plays a key role when implementing such a procedure. In literature, one important assumption is the independence between the claim frequency and the severity component and often they are considered separately when conducting model fitting and parameter estimations. Extending results demonstrated in Ni et al. (2014), we introduce in this work a dependence structure between the claim frequency and the claim severity component via a bivariate conjugate prior for the linear exponential familiy. We deduce correspondingly a bivariate a posteriori distribution for parameters related to the claim number and size, which in addition allows us to calculate an estimated net premium amount. A concrete example will also be presented with the bivariate prior conforming to a Bivariate mixed erlang distribution. We further use a set of automobile insurance data to first verify the dependence between the claim frequency and severity, and then fit the model and calculate the net premium amount numerically. Finally, a comparison with the independent case will be illustrated both theoretically and numerically.

Keywords. Models for insurance and finance, Bayesian methods, Bonus-Malus, Bivariate conjugate prior, Bivariate mixed Erlang

1 Introduction

This work is aimed at proposing a pricing model for Bonus-Malus systems in automobile insurance when the claim frequency and the claim severity component are considered to be dependent. A Bonus-Malus system is a merit rating system often seen in car insurance with the purpose of offering fair premiums among policyholders according to individual claim histories. A personal claim history is normally categorized into two components: the frequency and the severity which in literature have been assumed to be independent from each other. In this work, we suggest to establish a dependence structure via a bivariate conjugate prior when the model distributions are from the linear exponential family, e.g., Poisson, Exponential, etc. Explicit net premium functions could be obtained as the a posteriori estimation for the aggregate claims in a policy period.

The talk will begin by an introduction of modelling Bonus-Malus systems in car insurance industry. It will then be followed by a review of literature where normally an independence is assumed for the claim frequency and the claim severity component. For instance, Ni et al. (2014) did parameter estimations for those two components separately through a Bayesian approach and thus computing the net premium as a product of the estimations resulting from both components. Furthermore, the idea of employing a Bivariate conjugate prior originates from recent popularity of the use of multivariate Erlang mixtures. For more details, readers are referred to Willmot and Woo (2015) and Woo and Cheung (2013). Extending to a similar technique to that shown in Klugman et al. (2012) allows us to find the a posteriori distributions when model distributions are from the linear exponential family.

2 Main contribution

The first contribution of the work will concentrate on the derivation of the net premium amount when the dependence structure is taken into account. It extends the univariate conjugate priors (see e.g. Klugman et al. 2012) to a bivariate level and derives a bivariate a posteriori distribution for the parameters related to the claim frequency and the severity component, with which the a posteriori estimation for the aggregate loss in a single policy period, i.e., the net premium, could be obtained.

As a concrete example, we anlayse further the case when the bivariate conjugate prior follows a bivariate mixed erlang distribution. The resulting net premium function will be in a form of a non-separable double sum compared to the independent model. The reduction to the independent case would enable this double sum to be separable as a product of two sums.

In addition, we apply the model on a set of automobile insurance data employed in Ferreira and Minikel (2010). The initial step is to conduct a correlation test and to verify the presence of the dependence between the above mentioned two components. Subsequently, we reconstruct a parametric copula model using an Farlie-Gumbel-Morgenstern (FGM) copula and implement a fitting via the maximum likelihood theory, in order to find the starting parameter values. Furthermore, we attempt the goodness-of-fit test as introduced in Klugman and Parsa (1999). Using the estimated prior parameters, net premium amount could eventually be computed via a Bayesian adjustment. Results will be shown also in comparison with the independent case.

3 Conclusion

To conclude, the main contribution of this work is to build up a dependence structure between the claim frequency and the severity component. Such dependence is described using a bivariate conjugate prior for the linear exponential family models. Both analytical and numerical results are demonstrated in illustration of the model. The presence of the dependence was also verified by the given dataset. Comparing to the independent case, the dependent model provides a better fit to the data, and thus it would probably offer a more accurate pricing structure.

Bibliographie

[1] Ferreira, Jr. J. and Minikel, E. (2010), *Pay-As-You-Drive Auto Insurance in Massachusetts*, Conservation Law Foundation & Environmental Insurance Agency.
[2] Klugman, S.A., Panjer, H.H. and Willmot, G.E. (2012), Loss Models: From Data to Decisions, Fourth Edition, Wiley & Sons, Inc., Hoboken, New Jersey.
[3] Klugman, S. A., and Parsa, R. (1999), *Fitting bivariate loss distributions with copulas*, Insurance: mathematics and economics, 24(1), 139-148.

[4] Ni, W., Constantinescu, C., and Pantelous, A. A. (2014), *BonusMalus systems with Weibull distributed claim severities*, Annals of Actuarial Science, 8(02), 217-233.
[5] Willmot, G.E. and Woo, J.-K. (2015), *On some properties of a class of multivariate Erlang mixtures with insurance applications*, ASTIN Bulletin 45(1): 151-173.
[6] Woo, J.-K. and Cheung, E.C.K. (2013), *A note on discounted compound renewal sums under dependency*, Insurance: Mathematics and Economics 52(2): 170-179.